Interannual Variability and Predictability of 500 mb Geopotential Heights over the Northern Hemisphere

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ABSTRACT

The interannual variability of monthly mean 500 mb heights in a 15-year sample of observed data is compared to the variance expected from sampling errors associated with high-frequency fluctuations using the analysis of variance approach. The monthly mean "signal" stands out significantly from the "noise" over a substantial fraction of the Northern Hemisphere during the winter. The expected spectrum of variance at very low frequencies is assumed to be white at frequencies lower than (30 days)$^{-1}$ rather than the (96 days)$^{-1}$ cutoff used by Madden. This difference is justified by observing that the effective time between independent samples T_0 is relatively insensitive to changes in the maximum lag over which the local autocorrelation is integrated to calculate T_0. Further non-randomness in the variance of 500 mb heights is evidenced by the correlation between monthly mean height and contemporaneous daily variability.

1. Introduction

Interannual variability of monthly mean atmospheric states is caused both by internal dynamics and changes in external forcing. Since the boundary forcings due to anomalies in sea surface temperature, soil moisture, snow and sea ice, etc. change slowly compared to the atmospheric fluctuation, we will assume them to be "external" for discussion of predictability of monthly means. For longer periods these boundary forcings cannot be considered external because they themselves will be determined by their interaction with the internal dynamics. One of the important questions which has implications for predictability is: What are the relative roles of internal dynamics and boundary forcings in explaining the observed interannual variability? The underlying assumption is that changes due to internal dynamics are mostly unpredictable, whereas those due to boundary forcings are potentially predictable, and therefore, if observed interannual variability is significantly larger than what could be caused by internal dynamics alone, there will be potential for predictability of time averages.

We can visualize a hypothetical situation in which sea surface temperature, soil moisture, sea ice, and snow, etc. are constant with time, and atmospheric flows evolve only due to dynamical instabilities and nonlinear interactions among different space and time scales, including interactions of fluctuating zonal winds with orography and stationary diabatic forcings due to land–sea contrast. If we consider a long time evolution of such a flow, the monthly means will be different for different 30 day samples. Variability of such 30 day means can be considered similar to what Madden (1976) has termed as "natural variability."

Madden (1976) used the observed data to calculate the "natural variability," for monthly mean sea level pressure over the Northern Hemisphere. He calculated the observed spectrum and used a low frequency white noise (LFWN) extension for frequencies lower than (96 days)$^{-1}$ to estimate the standard error of monthly means. He concluded that the observed total variability was not much larger than his estimate of the natural variability in middle latitudes, and therefore that the potential predictability of monthly means was slight.

Straus and Halem (1981) tested the separability of internal dynamics and boundary forcing variability by comparing the local autocovariance functions (acf's) of sea level pressure and surface temperature calculated from observed data and from an ensemble of one-month long general circulation model runs with fixed boundary conditions. The observed and model-generated sea level pressure acf's were not significantly different, indicating support for the separability assumption on a time scale of just one month. However, significant differences were seen in the comparison of temperature acf's.

Shukla (1983) pointed out that the method used by Madden (1976) tends to overestimate the natural variability and therefore underestimate the potential predictability. In particular, Shukla criticised Madden's assumption of the potentially predictable climatic signal to be only the variance in frequencies lower than (96 days)$^{-1}$ and above white noise. He

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argued that predictable changes due to boundary forcing could occur over periods shorter than a season (such as one month). Besides, Madden's conclusions do not apply for dynamical predictability of a given initial state.

In this paper we will employ the analysis of variance approach to test whether January means are different from one year to another. We would show that the present calculation is equivalent to finding the ratio of observed variance and standard error of monthly means.

The variability within a month will be assumed to be completely determined by the internal dynamics and the observed total variability will be considered to be due to the combined effects of internal dynamics and boundary forcings. If the latter is significantly larger than the former, it will be considered as evidence of potential predictability.

Our approach differs from Madden's in two ways. First, we will explicitly calculate the autocorrelation function at each gridpoint for lags as long as the climatic sample time to estimate the degrees of freedom in our variability computations; second, we will use a maximum lag of 30 days to calculate this estimate, implicitly assuming a LFWN extension at that point rather than at 96 days. With these modifications, our calculations yield somewhat more optimistic estimates of potential predictability.

Even so, we believe that this calculation might still represent an underestimate of predictability. Recent studies indicate that the null hypothesis underlying this type of statistical calculation, the assumption of absolutely no dynamical predictability on monthly time scales, may not be realistic. We shall discuss this possibility at the conclusion of the paper.

2. Data base

The data set used for this study contains daily maps of 500 mb geopotential height for 15 January and 15 July months (1963–77). The data was derived from NMC final analyses and obtained from the NCAR data archive. Each map consists of a 4° latitude by 5° longitude grid covering the Northern Hemisphere from 22 to 90°N. Temporal linear interpolation was performed to replace a few missing or obviously erroneous grids.

3. The climatic signal compared to high-frequency noise

In this section, we will calculate estimates of the ratio of observed interannual variability to the natural variability for the months of January and July.

For each gridpoint, consider the time series of daily 500 mb heights ϕ_{ij} for a certain month of the year, sampled for several years. Here the first subscript i represents the day, and the second subscript j represents the month. We then calculate the set of monthly means M_j. Each M_j defines the climatic state of the 500 mb height field for month j, and is assumed to be determined, at least in part, by slowly varying boundary forcing. The variance $\sigma_M{}^2$ of the time series M_j describes the interannual variability of monthly means.

$$\sigma_M{}^2 = (J-1)^{-1} \sum_j (M_j - \bar{M})^2 = (n_j(J-1))^{-1}$$
$$\times [n_j{}^{-1} \sum_j (\sum_i \phi_{i,j})^2 - (Jn_j)^{-1}(\sum_i \sum_j \phi_{i,j})^2]$$
$$M_j = n_j{}^{-1} \sum_i \phi_{i,j}, \quad \bar{M} = J^{-1} \sum_j M_j,$$

where n_j is the number of days in jth month ($n_j = 31$ for each j), and $J = 15$ is the number of months.

We have calculated the F ratio using the daily geopotential heights (ϕ).

$$F(\nu_1, \nu_2) = \frac{\sum_j \dfrac{(\sum_i \phi_{ij})^2}{n_j} - \dfrac{(\sum_j \sum_i \phi_{ij})^2}{N}}{\sum_j \sum_i \phi_{ij}^2 - \sum_j \left[\dfrac{(\sum_i \phi_{ij})^2}{n_j}\right]} \cdot \frac{\mathrm{dof}_i}{\mathrm{dof}_j}, \quad (1)$$

where $N = n_j J = 31 \times 15$. To calculate the degrees of freedom dof_i, we first calculate the characteristic time (T_0) between independent samples in the daily time series of 500 mb heights.

The value of dof_i is then given as:

$$\nu_2 = \mathrm{dof}_i = \{[(n_j J)/T_0] - J\} = 15[(31/T_0) - 1],$$
$$\nu_1 = \mathrm{dof}_j = J - 1 = 14. \quad (2)$$

It should be pointed out that an alternative procedure for calculating the ratio of observed variances of monthly means ($\sigma_M{}^2$) and standard error of the monthly means ($\sigma_E{}^2$), would be:

$$\hat{F} = \frac{\sigma_M{}^2}{\sigma_E{}^2}, \quad (3)$$

where $\sigma_E{}^2 = \sigma^2 T_0/n_j$ and σ^2 is the variance of daily height values which can be assumed to be normally distributed (White, 1980).

$$\sigma^2 = J^{-1} \sum_j \sigma_j{}^2$$
$$= (n_j(J-1))^{-1}[\sum_j \sum_i \phi_{i,j}^2 - n_j{}^{-1} \sum_j (\sum_i \phi_{i,j})^2].$$

The ratio of F and $\hat{F}$ is given as

$$\frac{F}{\hat{F}} = \frac{(n_j - T_0)}{(n_j - 1)}.$$

This would imply that the values of F given by (1), which we present in this paper have been underestimated by ~7% for $T_0 = 3$. The characteristic time T_0 is defined (Leith, 1973) as

$$T_0 = 1 + \sum_{\tau=1}^{T} 2\left(1 - \frac{\tau}{T}\right)r(\tau), \qquad (4)$$

where $r(\tau)$ is the autocorrelation of 500 mb heights at lag τ. To obtain reliable estimates of autocorrelation at long lags, data for the entire winter (December, January and February) and summer (June, July and August) were used. The local autocorrelations necessary for this estimate were calculated and discussed in detail by Gutzler and Mo (1983).

The estimated sampling times for winter and summer data, with T set to 30 days, are shown in Fig. 1. Wintertime values (Fig. 1a) range from minima of less than three days over the east coast of Asia and much of North America to maxima of longer than one week over the North Atlantic and northern Siberia. These estimates are quite close to those by Madden (1976) and Stefanick (1981) with the exception of high values over central Asia. It was pointed out by Trenberth and Paolino (1980) that the monthly mean sea level pressure data had major problems over Asia and therefore the Asian values in all the calculations should be interpreted with caution.

Summertime estimates exhibit somewhat less geographical coherence. Minima of about two days exist off the coasts of California and Morocco and over the Mediterranean Sea. Other areas where the autocorrelation decays quickly include Scandinavia, northern Canada, northern Japan and the Bering Sea. Maxima are found over the North Pacific, the Saharan region and the pole. The band of maxima extending from Japan eastward over the Pacific is consistent with the results of Madden and with the observed maxima of low frequency variability.

We tested the sensitivity of T_0 to changes in the maximum lag T used for the sum in Eq. (4). Wintertime estimates of T_0 were very stable for values of T between 15 and 30 days. For the summer, T_0 increases with increasing T over the subtropics, because the autocorrelation decays very slowly and remains positive for long lags (Gutzler and Mo, 1983). The stability of T_0 with respect to changes in T suggests that we have reasonably represented the low frequency end of the variance spectrum.

Plots of F for the months of January and July are shown in Fig. 2. Values of F greater than 2.0 represent a signal that stands out significantly (with 97.5% confidence) above the noise, based on F-test significance tables assuming degrees of freedom of 14 and 50. Areas where F exceeds this threshold are indicated in Fig. 2. During the winter, these regions include most of the Pacific Ocean, central Asia, northern Europe, the polar area, the far northern Atlantic, and the eastern United States. Here F is greater than 2 over substantial areas of the hemisphere, particularly in the winter, even in middle latitudes where day-to-day variability is largest. The fractional area over which F exceeds 2.0 is much greater than the area expected by chance, even considering the spatial correlation exhibited by the 500 mb height field.

The F values are generally smaller in the summer, with the exception of the subtropical oceans, where the signal-to-noise ratio is much higher compared to winter.

We have repeated the calculation of F using NMC sea level pressure analyses for the same period of record (not shown); the results were quite similar to the 500 mb height statistics presented here. Madden did not analyze large areas of the hemisphere where his long data record contained temporal discontinuities. Even the relatively short time series used here might contain inhomogeneities, due to changes in the NMC analysis scheme and in the distribution of observing stations on which the analysis is based, which would make the time series nonstationary.

However, several important procedural differences probably contributed to the increased predictability implied by our calculations. Madden's definition of climatic noise included all fluctuations with periods of 96 days or less and white noise beyond that. We suggest that 30 days is a more appropriate cutoff. The changing boundary conditions and low frequency internal variability responsible for the climatic signal may (predictably) change on time scales of a month. The importance of this difference can be seen in Madden's Table 1, in which it is seen that nearly all the unpredictability of the monthly means come from fluctuations with periods of 48 days or longer. Moreover, it is quite reasonable to expect that persistence and autocorrelation at 500 mb would be different from that at the surface.

Further evidence of non-randomness in the high frequency variability is shown in Fig. 3. In these plots, the time series of the departure from the seasonal cycle of monthly means M_j was correlated with the contemporaneous daily variance σ_j at each gridpoint. To increase the significance of the correlations, Fig. 3a was calculated using data for all winter months (December, January, February), and Fig. 3b using all summer months (June, July, August). Correlations with magnitudes greater than ~0.4 are then significant at the 99% level. In wintertime, anomalously high daily variability is associated with positive monthly mean height anomalies over the northern oceans and negative monthly mean height anomalies over the subtropics.

4. Discussion

The results presented in the previous section seem to imply more encouraging prospects for climate predictability than could be drawn from Madden's results. However, results of this and Madden's study

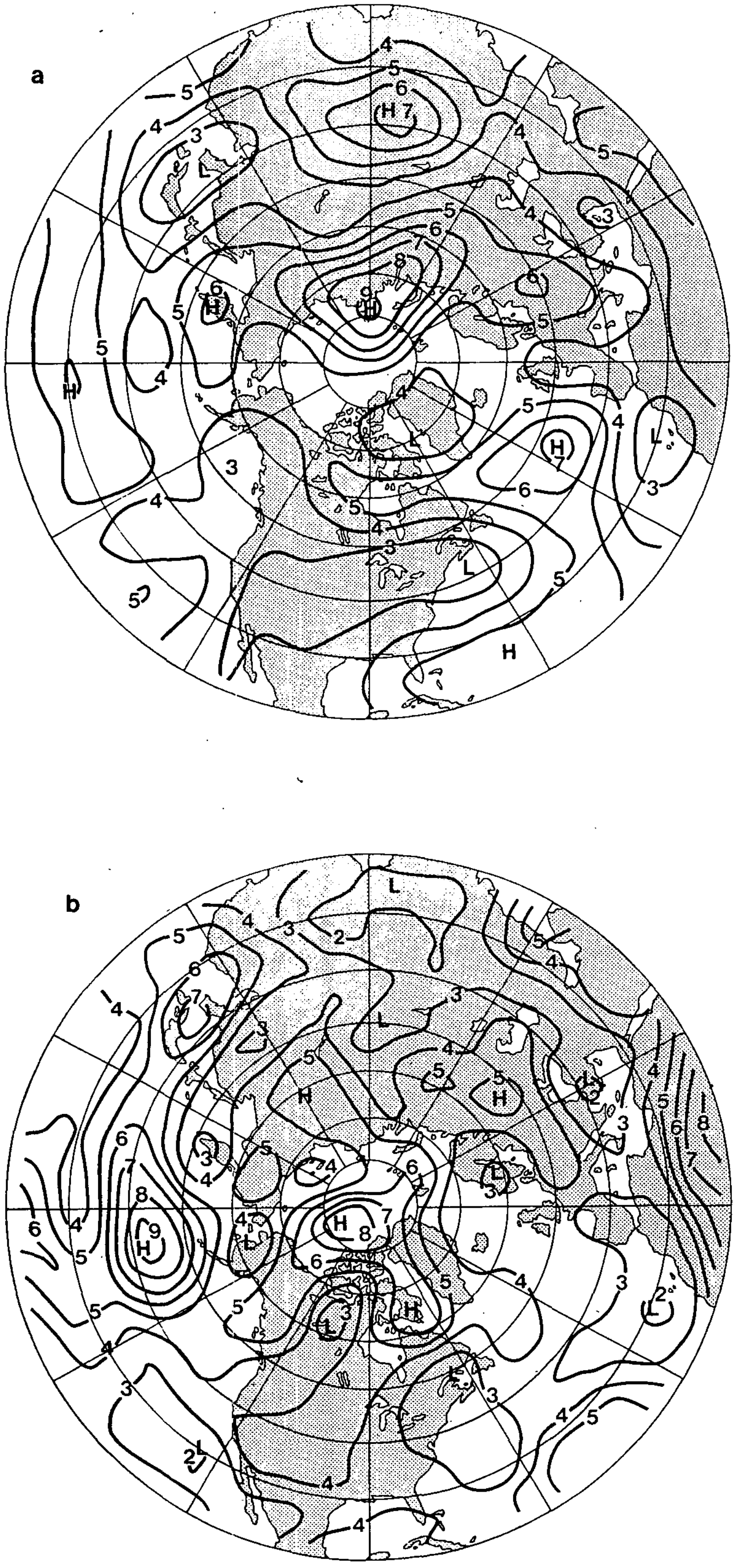

FIG. 1. Estimated time (days) between independent samples of the 500 mb height field for (a) winter, (b) summer. Contour interval 1 day.

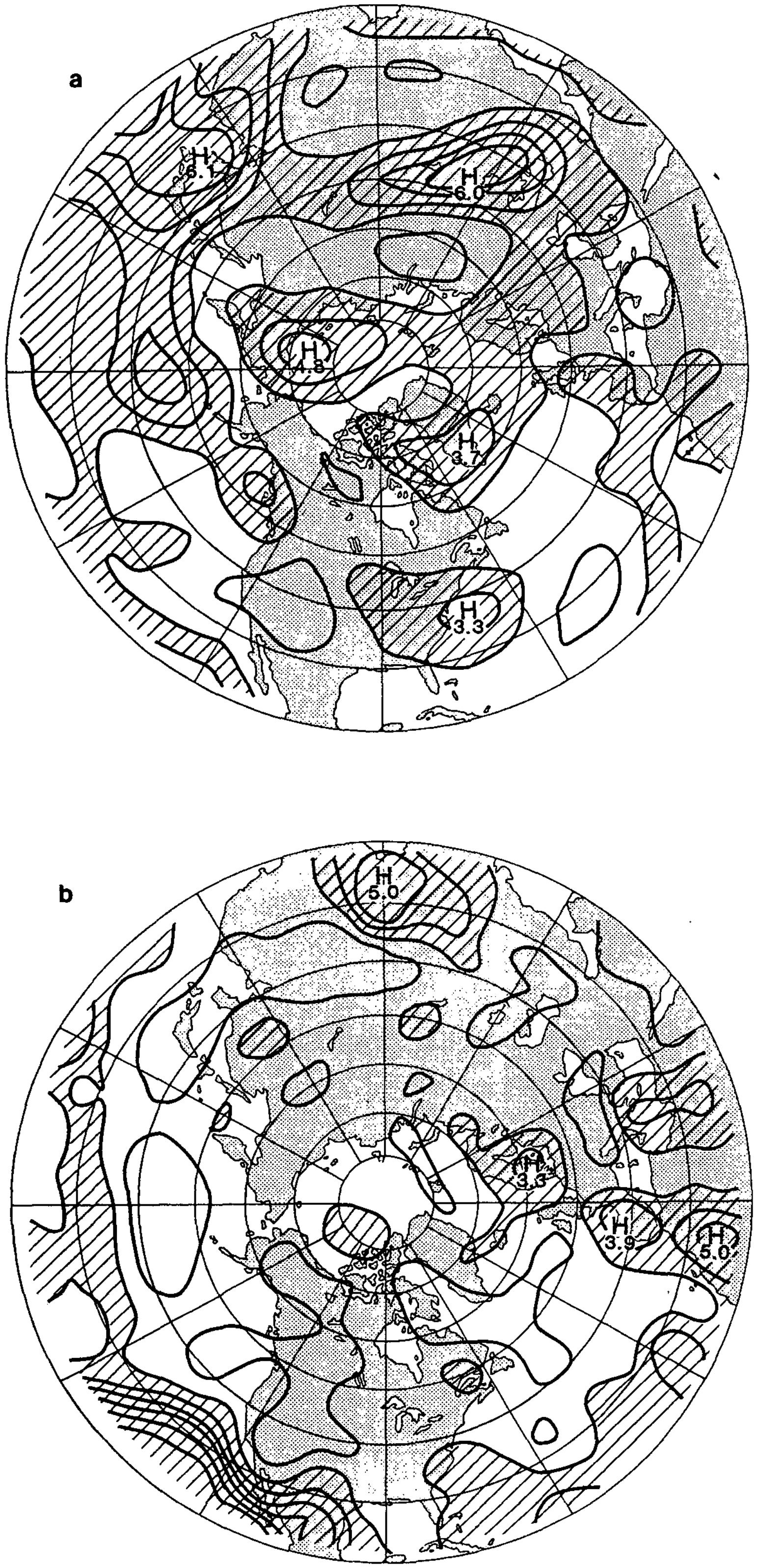

FIG. 2. *F*-statistic values representing the ratio of interannual to intraseasonal variability for (a) January, (b) July. Contour interval 1.0. Stippled areas indicate *F* values greater than 2.0, which are significant at greater than the 97.5% level.

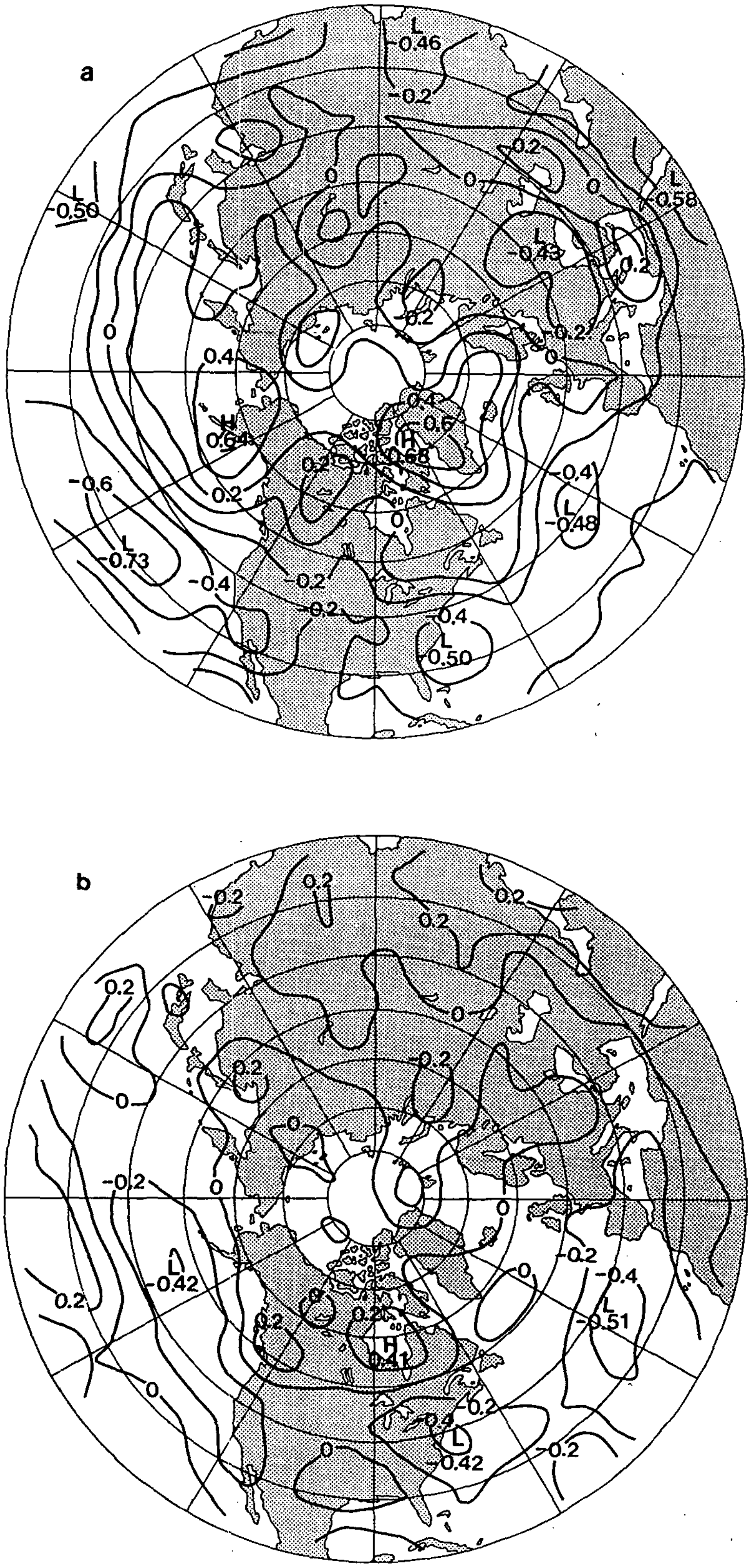

FIG. 3. Correlation coefficient between monthly mean 500 mb height and variance of daily 500 mb height during the same month, for (a) winter months (December, January, February), (b) summer months (June, July, August). Contour interval 0.2.

are not relevant for the dynamical predictability of a given initial state. It is important to keep in mind that both studies provide only statistical estimates based on a very pessimistic null hypothesis, which was the complete lack of dynamical predictability. Recent evidence indicates that this hypothesis may be unrealistic.

The definition of climatic noise used in the F ratio includes the assumption that all high-frequency variability is associated with deterministically unpredictable weather. This supposition may not be correct for two reasons. First, the high-frequency variability may not be completely independent of the time-mean state, as suggested in Fig. 3. Second, the weather fluctuations themselves may be predictable on a time-average basis. Shukla (1981) showed that even with constant boundary forcing, the low-frequency signal associated with the persistence and evolution of long waves should be strong enough to permit skillfull monthly mean forecasts despite the noise created by instabilities in the flow.

Dynamical predictability might also be enhanced by the tendency for certain climatic states to be more predictable than others. Predictability as defined using the F test refers to an average over many initial states, so that F would be underestimated if some initial states were known to be more persistent than others. Bengtsson (1981) has documented a successful numerical long-range forecast for a blocking situation. Blocking occurs most frequently in precisely those regions where σ_j^2 is a maximum (Dole, 1982; Shukla and Mo, 1983). It seems quite likely that a realistic dynamical model could predict the evolution of the low-frequency long-wave components of certain flow configurations for up to one month so that prediction of monthly means may be reasonable.

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